Problems on Inference-I

1. For double genetically data with some value of π, for both parents, following distribution is obtained:

	D1D2	D1R2	D2R1	R1R2
Frequency:	190	36	34	27
Probability:	$\frac{2+p}{4}$	$\frac{1-p}{4}$	$\frac{1-p}{4}$	$\frac{p}{4}$

where $p=(1-\pi)^2$. Find Maximum Likelihood Estimate of π and estimate its standard error.

2. A random sample of size 20 is drawn from a population with the probability density function $f(x,\theta)=\frac{1}{\theta}e^{-\frac{x}{\theta}}; x,\theta>0$ and the sample mean comes out to be 12.6. Find MLE of θ. How do you modify the estimate if 2 sample observations are known to exceed value 60 only? Also how do you modify in drawing the sample observation exceeding 60 is rejected.

3. Following data represent a random sample of size from the Cauchy population with the probability density function $f(x,\theta)=\frac{1}{\pi}.\frac{1}{1+(x-\theta)^2}; -\infty<x,\theta<\infty.$ Find out the MLE of θ. The observations are 3.7807, 2.9957, 5.2043, 4.8993, 2.6874, 4.9557, 4.9367, 3.4996, 3.1674.

Without assuming any distribution, find out nonparametric estimate of mean and variance functional.

4. A random variable X takes values 0, 1, 2 with respective probabilities $\frac{\theta}{4N}+\frac{1}{2}\left(1-\frac{\theta}{N}\right)$, $\frac{\theta}{2N}+\frac{\alpha}{2}\left(1-\frac{\theta}{N}\right)$ and $\frac{\theta}{4N}+\frac{1-\alpha}{2}\left(1-\frac{\theta}{N}\right)$, where N=25 is a known number and α, θ are unknown parameters. If 75 independent observations on X yielded the values 27, 38, 10 respectively, estimate θ and α by method of moments.

5. Consider the problem of point estimation of θ in $N(\theta,1)$. Given that θ belongs to [-1, 1]. On the basis of a sample of size n, the following estimator has been defined.

T=-1 if $\bar{X} < -1$

= $\bar{X}$ if $-1 \le \bar{X} \le 1$

= 1 if $\bar{X} > 1$

$\bar{X}$ being sample mean. Assuming (i) squared error loss and (ii) absolute error loss draw the risk curve of $\bar{X}$ and T over the range $\theta \in [-1,1]$ on the same graph paper and comment. Take n=10.

6. The total amount of claims for each year from a portfolio of five insurance policies over *t* years were found to be $X_1, X_2, \ldots, X_t$. The insurer believes that the annual claims have a normal distribution with mean μ and variance σ^2, where μ is unknown. The prior distribution of μ is assumed to be normal with mean γ and variance η^2.

(i) Derive the posterior distribution of μ.

(ii) Using the posterior distribution found in (i), write down the Bayesian point estimate of μ under the quadratic (squared error) loss function.

(iii) Show that the answer in (ii) can be expressed in the form of a credibility estimate, and derive the credibility factor.
(iv) If one uses the all-or-nothing loss function, can the corresponding Bayesian estimate of μ be written as a credibility factor? Explain.
Let $(X_1, X_2, \ldots, X_t)=(1050, 1175,1100,1200,1150)$, $\sigma^2=400$, $\gamma=1110$, $\eta^2=256$. Evaluate (ii) and (iii). Draw the risk function of the Bayes estimate. Also calculate Bayes risk of the estimate.